Investigation of Multi-Input, Multi-Output (MIMO) Random Control Applied to Direct Field Acoustic Testing

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Outline

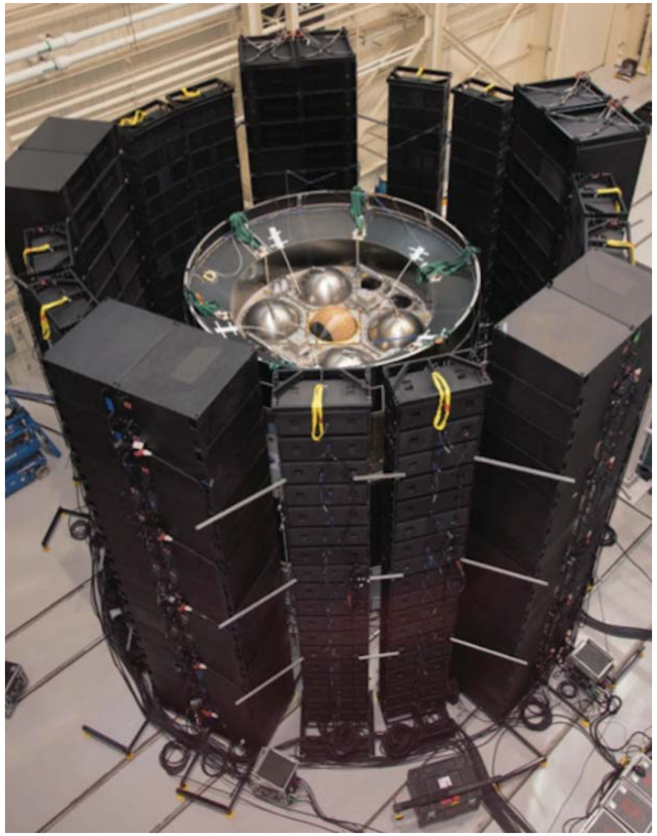
- **Motivation**
- DFAT versus Reverb Test results
- MIMO Control Theory
- Numerical Simulation of DFA Test
- Alternative DFAT & MIMO Control Configurations
- What we learned

Qualification of Direct Field Acoustic Testing for NASA Manned Space Missions

Reverberant Chamber Testing

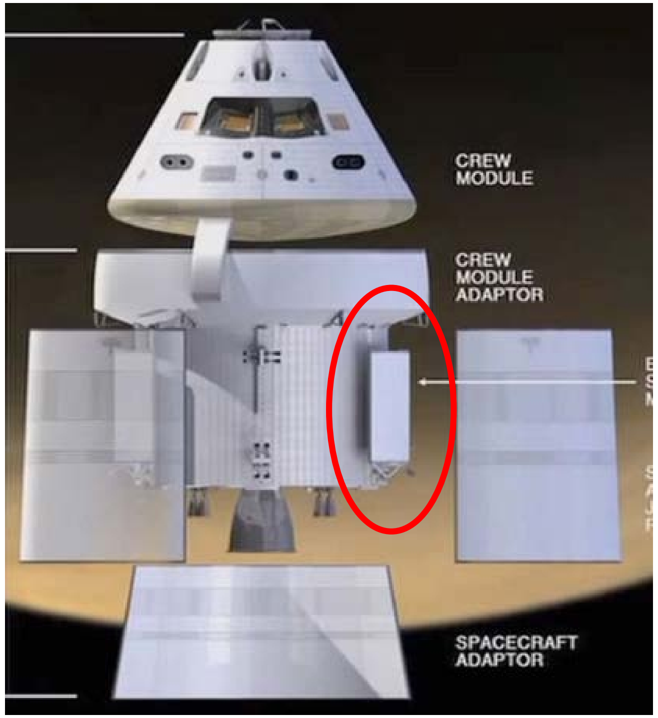
$$G_{pp}\left(\mathbf{x},\mathbf{x}';\omega\right)=\left\langle G_{pp}\left(\omega\right)\right\rangle \frac{\sin\left(k_0\left|\mathbf{x}-\mathbf{x}'\right|\right)}{k_0\left\|\mathbf{x}-\mathbf{x}'\right\|}$$

Direct Field Acoustic Testing (DFAT)



Vibration response under random acoustic loading

QUARTUS ENGINEERING



- **FE / BEM**

$$G_{vv}(\mathbf{x},\omega)=\sum_r \frac{\omega^4\psi_r^2(\mathbf{x})S_{ff,r}(\omega)}{g^2m_r^2\left|\omega_r^2(1+j\eta_r)-\omega^2\right|^2} \qquad g^2/Hz$$

$$S_{ff,r}(\omega)=\iint_A \psi_r(\mathbf{x})G_{pp}(\mathbf{x},\mathbf{x}';\omega)\psi_r(\mathbf{x}')\,d\mathbf{x}\,d\mathbf{x}'$$

$$S_{ff,r}^{REV}(\omega)=\left\langle G_{pp}(\omega)\right\rangle \underbrace{\iint_A \psi_r(\mathbf{x})\frac{\sin k_0\left|\Delta\mathbf{x}\right|}{k_0\left|\Delta\mathbf{x}\right|}\psi_r(\mathbf{x}')\,d\mathbf{x}\,d\mathbf{x}'}$$

$$=\left\langle G_{pp}(\omega)\right\rangle \quad j_r^2(\omega)$$

- **SEA**

$$\left\langle G_{vv,\Delta\omega}\right\rangle=\frac{\omega\pi A^2}{2g^2m^2}\frac{n_{\Delta\omega}}{\overline{\eta}_{\Delta\omega}}\left\langle G_{pp,\Delta\omega}\right\rangle \overline{j}_{\Delta\omega}^2 \qquad g^2/Hz$$

DFAT vs Reverberation Chamber Testing: **Qualification Metrics**

SOUND PRESSURE

1. Third octave, RMS spectrum level	±3 dB
2. Spatial uniformity	±2 dB
3. Spatial correlation	TBD

SPACECRAFT VIBRATION

4. Third octave, RMS spectrum level	±3 dB

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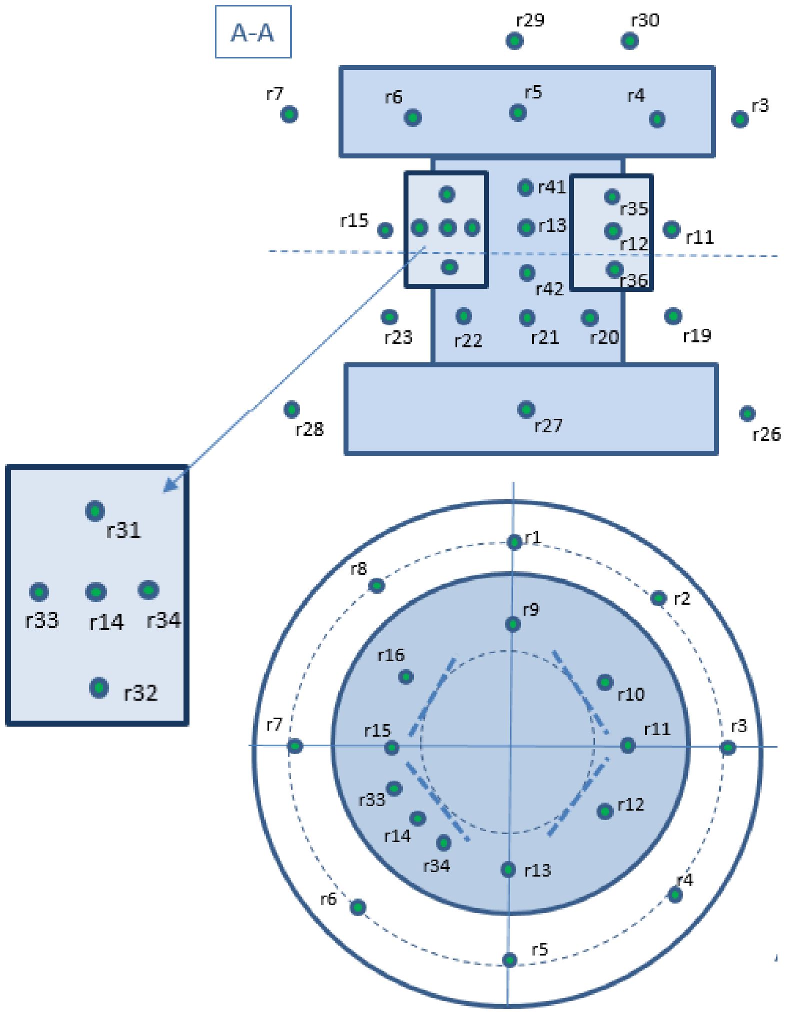
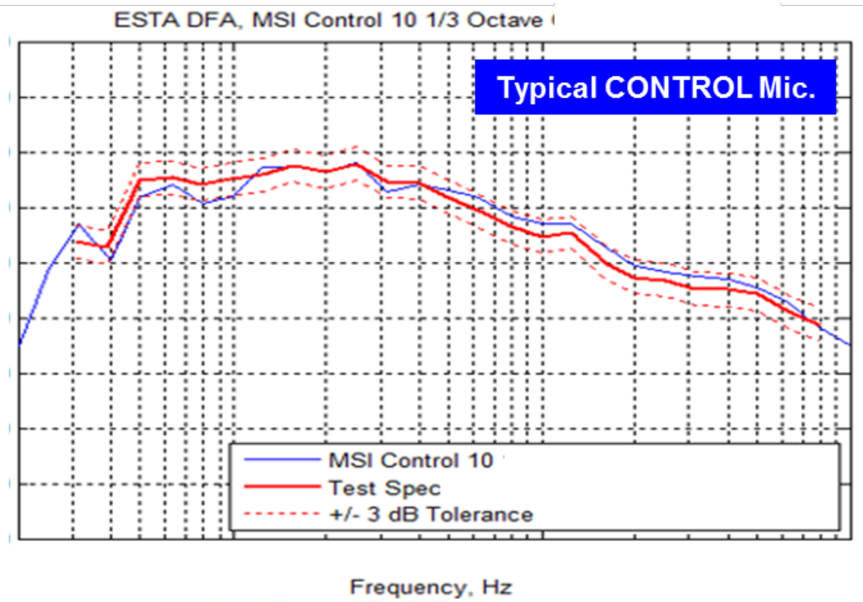
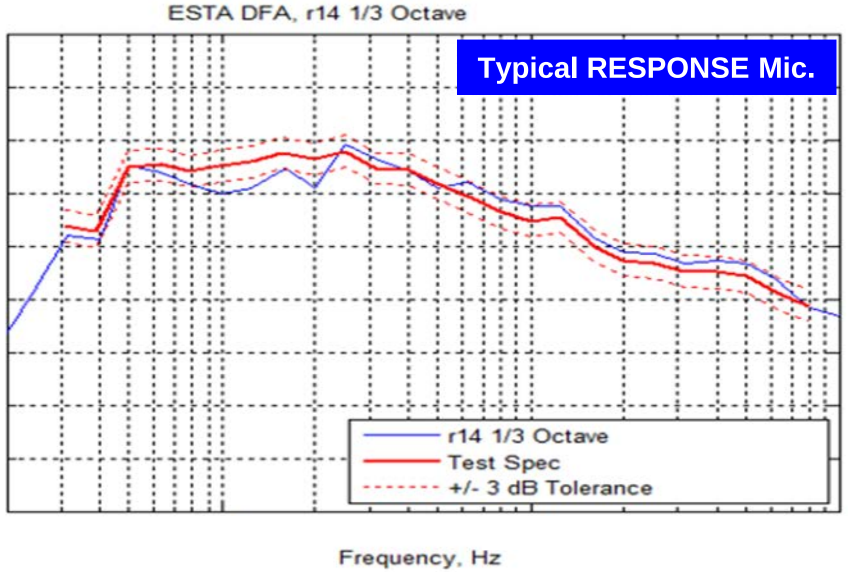
Test Results - **Acoustic field**
DFAT SPL versus Test Spec.

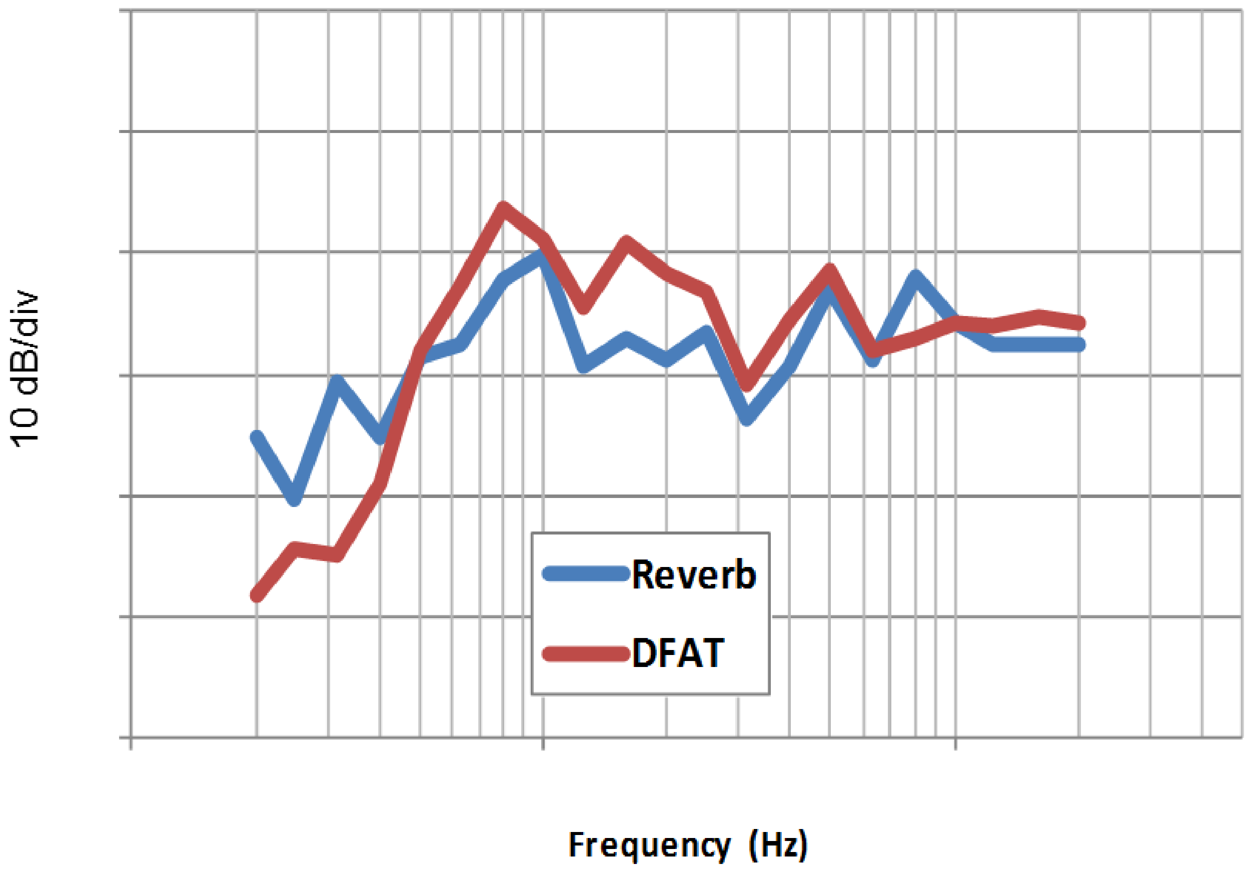






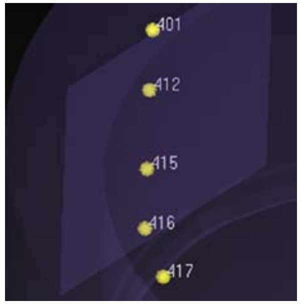
Test Results – **Spacecraft Vibration**
Reverb Chamber versus DFAT

Spacecraft Structure – Sample <u>Normalized</u> Vibration Response



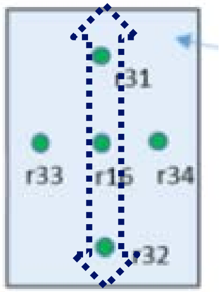
Test Results – **Spatial Correlation**
Reverb. Chamber versus DFAT

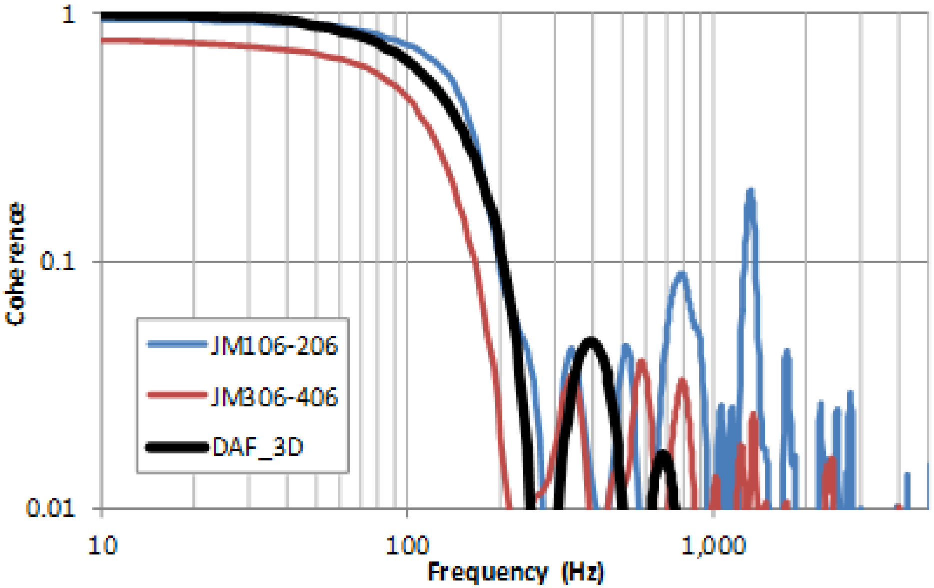
Reverberation Chamber Test

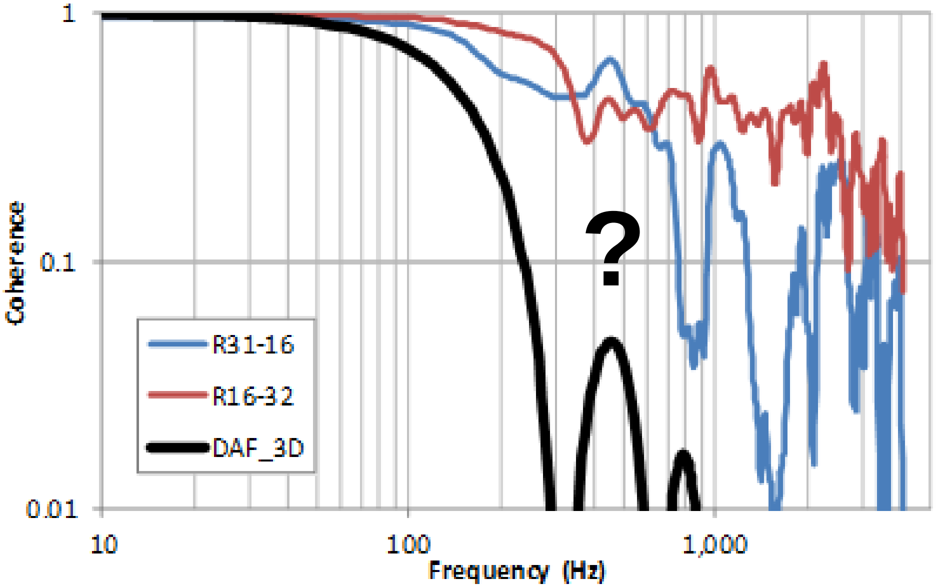


$$\gamma^2_{DAF,3D} = \frac{\left|G_{pp}\left(\mathbf{x},\mathbf{x}';\omega\right)\right|^2}{G_{pp}\left(\mathbf{x},\omega\right)G_{pp}\left(\mathbf{x}',\omega\right)} = \left[\frac{\sin\left(k_0\left|\Delta\mathbf{x}\right|\right)}{k_0\left|\Delta\mathbf{x}\right|}\right]^2$$

Direct Field Acoustic Test







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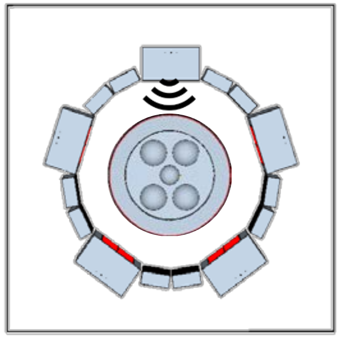
MIMO Random Control Theory

- Wave6 BEM solves for deterministic frequency response between input voltage (velocity) and output sound pressure

$$\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_r \end{Bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & & h_{2m} \\ \vdots & & \ddots & \\ h_{r1} & h_{r2} & \cdots & h_{rm} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{Bmatrix}$$



(Eq. 1) $\mathbf{p} = \mathbf{H}\mathbf{v}$

- Random drive signals result in random pressures which can only be quantified statistically - autospectrum G_{pp}, coherence γ^2_{ij} and phase φ_{ij} depends on:
  - BOTH cross spectrum of input voltages (velocities) AND frequency response functions

(Eq. 2)

$$\begin{aligned} \mathbf{G}_{pp} &= E\left[\mathbf{p}\,\mathbf{p}^{*T}\right] \\ &= E\left[\mathbf{H}\mathbf{v}\left(\mathbf{H}\mathbf{v}\right)^{*T}\right] \\ &= \mathbf{H}\,\mathbf{G}_{vv}\mathbf{H}^{*T} \\ &= \begin{bmatrix} G_{11}(\omega) & G_{12}(j\omega) & \cdots & G_{1s}(j\omega) \\ G_{21}(j\omega) & G_{22}(\omega) & & G_{2s}(j\omega) \\ \vdots & & \ddots & \\ G_{r1}(j\omega) & G_{r2}(j\omega) & \cdots & G_{rs}(\omega) \end{bmatrix} \end{aligned}$$

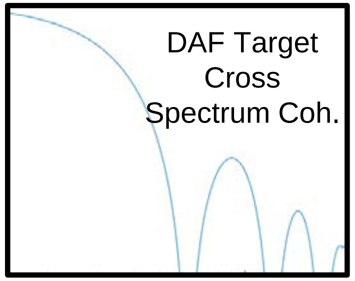
MIMO Random Control for Diffuse Acoustic Field - I

- For DAF we can fully define the *required* $G_{pp}(j\omega)$ pressure cross spectrum matrix

$$\mathbf{G}_{rs}(j\omega)=\langle G_{pp}(\omega)\rangle\begin{bmatrix}1 & \gamma_{12}^2(j\omega) & \cdots & \gamma_{1s}^2(j\omega)\\ \gamma_{21}^2(j\omega) & 1 & & \gamma_{2s}^2(j\omega)\\ \vdots & & \ddots & \\ \gamma_{r1}^2(j\omega) & \gamma_{r2}^2(j\omega) & \cdots & 1\end{bmatrix}$$

(Eq. 3)

$$\gamma_{rs}^2(\mathbf{x}_r,\mathbf{x}_s,\omega)=\left[\frac{\sin\left(k_0\left|\mathbf{x}_r-\mathbf{x}_s\right|\right)}{k_0\left|\mathbf{x}_r-\mathbf{x}_s\right|}\right]^2$$



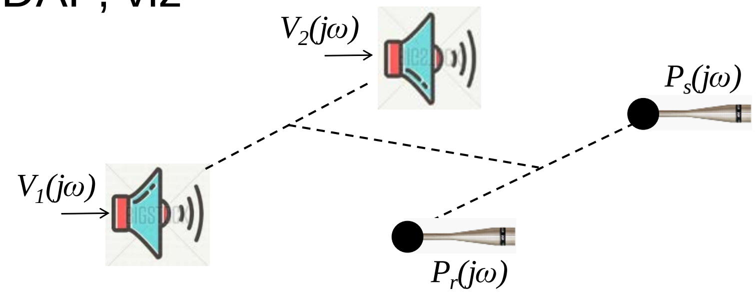
- And use inverse of the wave6 frequency response function matrix $H_{rm}(j\omega)$ to define the required cross spectrum of input voltages (velocities)

(Eq. 2.1)

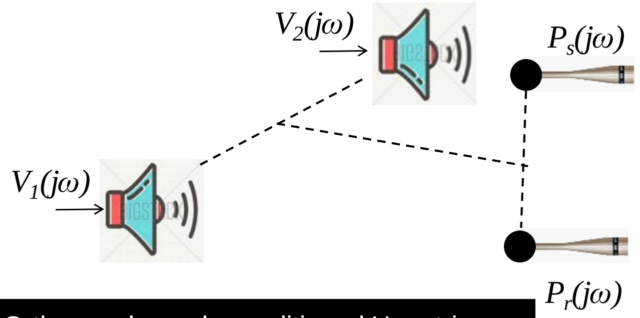
$$\mathbf{G}_{vv}=\mathbf{H}^{-1}\,\mathbf{G}_{pp}\left(\mathbf{H}^{*T}\right)^{-1}$$

MIMO Random Control for Diffuse Acoustic Field - II

- HOWEVER for certain physical configurations of audio sources and control microphones it may be physically impossible for the frequency response functions to support the mixing of response pressures required to achieve a DAF; viz



Aligned, well-conditioned H matrix



Orthogonal, <u>poorly</u> conditioned H matrix

  – In which case, the H matrix may be singular (not invertible)

- Physically, this means that some *impossibly large drive voltages* would be required to achieve the specified DAF

MIMO Random Control for Diffuse Acoustic Field - III

- Furthermore, a MIMO controller can utilize a **rectangular control** strategy
- **# Outputs > # Inputs**, therefore there is no “exact” solution

$$\left|\left|\boldsymbol{H G_{vv} H^{*T} - G_{pp}}\right|\right| \neq \boldsymbol{0}$$

… the result is a “least squares” solution

$$\boldsymbol{G_{vv} = H^{+} G_{pp} \left(H^{*T}\right)^{+}}$$

- Where the pseudoinverse is derived from SVD of H

$$\boldsymbol{H = U W V^{T}} \quad \Rightarrow \quad \begin{aligned} \boldsymbol{H^{+}} &= \boldsymbol{V^{T} W^{-1} U} \\ &= \boldsymbol{\left(H^{T} H\right)^{-1} H^{T}} \end{aligned}$$

$$\boxed{\boldsymbol{G_{vv} = \left(H^{T} H\right)^{-1} H^{T} G_{pp} H^{*} \left(H^{*} H^{*T}\right)^{-1}}}$$

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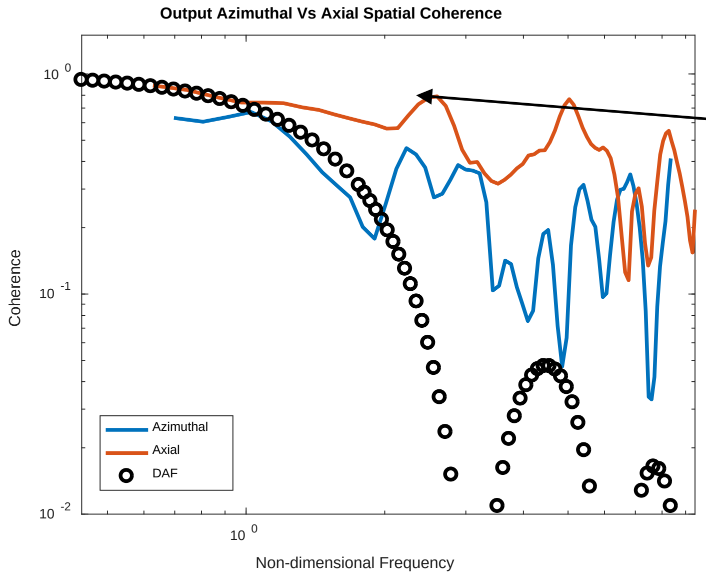
eSTA DFAT Experimental Data

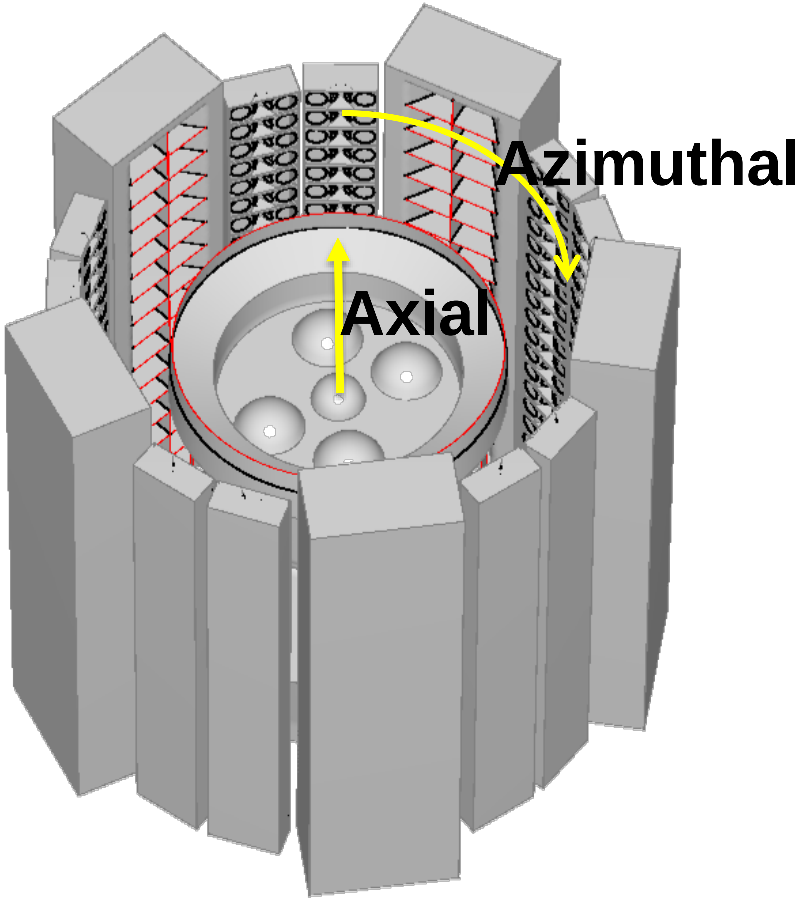
- Experimental data shows axial cross spectra does not approach Diffuse Acoustic Field

Test Data Spatial Coherence



Vertical stacks have the same input along the entire height which inhibits axial decoupling

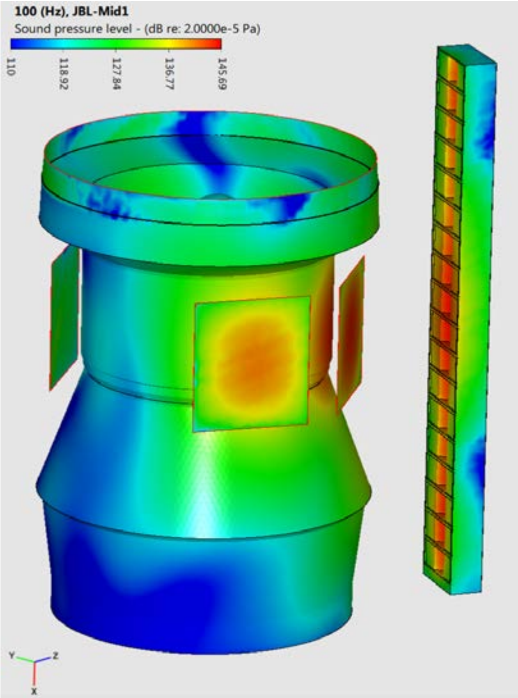
DFAT Loud Speaker Configuration

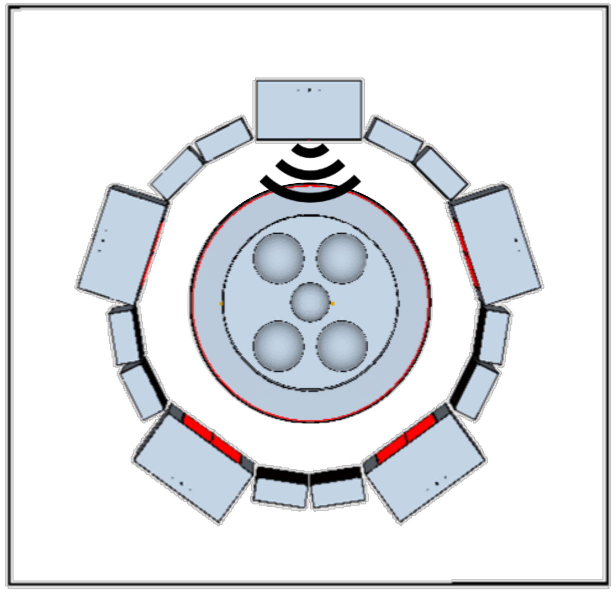


BEM Scattering Simulation

- Scattering simulations include the effects of sound reflecting off of spacecraft and speaker surfaces

Frequency Response Function Evaluation





$$\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_r \end{Bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & & h_{2m} \\ \vdots & & \ddots & \\ h_{r1} & h_{r2} & \cdots & h_{rm} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{Bmatrix}$$

$$\mathbf{p} = \mathbf{H}\mathbf{v}$$

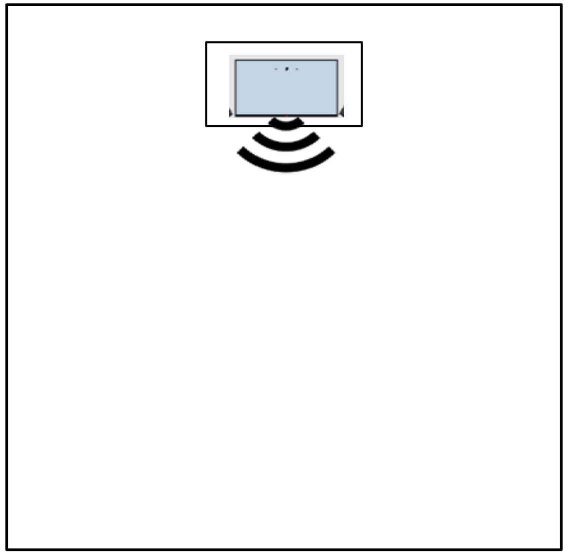
- *FRFs are evaluated one speaker at a time*
- *FRFs include effects of sound bouncing off remaining geometry*

BEM Direct Field Simulation

- Direct field simulations assume that effects of scattering are negligible with respect to direct speaker output

Frequency Response Function Evaluation

Direct Field Simulation



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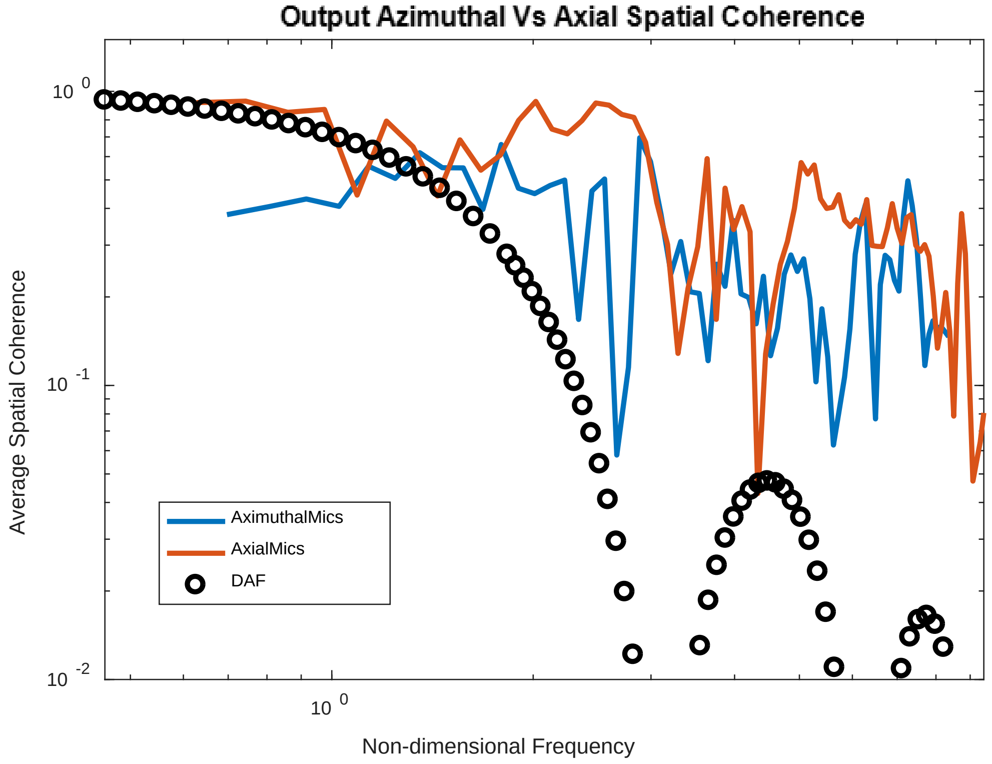
$$\mathbf{p} = \mathbf{H}\mathbf{v}$$

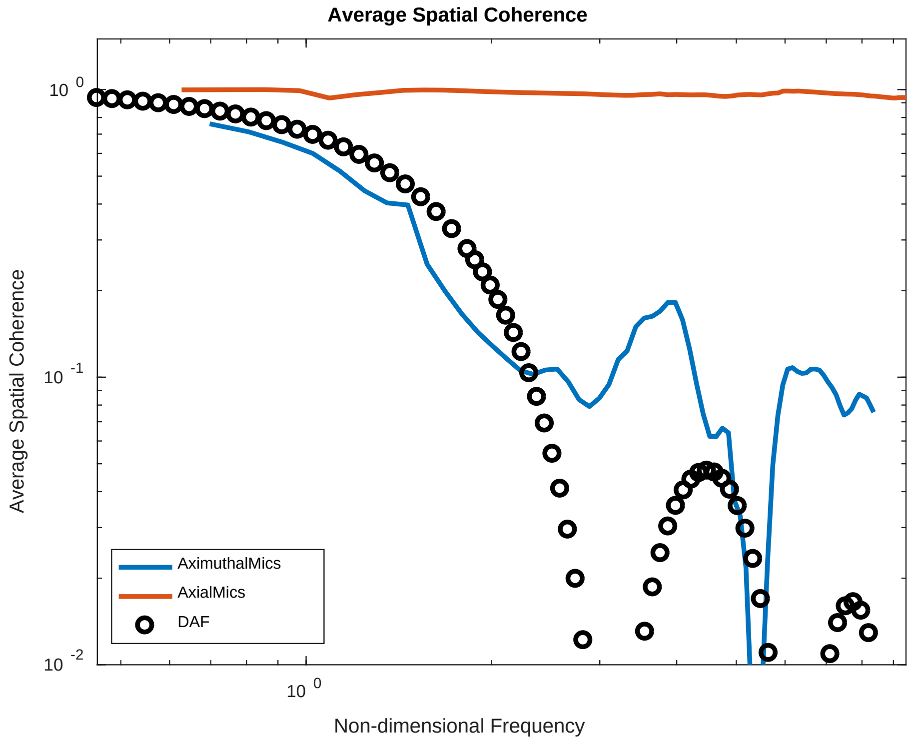
- *FRFs are evaluated one speaker at a time*
- *FRFs neglect effects of sound bouncing off remaining geometry*

BEM Simulation versus Test
DFAT Spatial Correlation

Scattering Simulation



Direct Field Simulation



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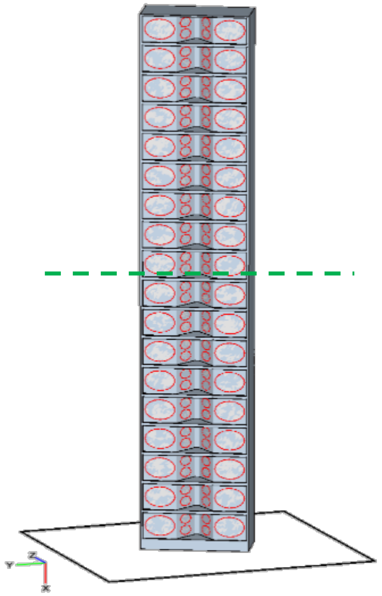
Split Simulation

- Dividing the speakers into 2 partitions (vertically)

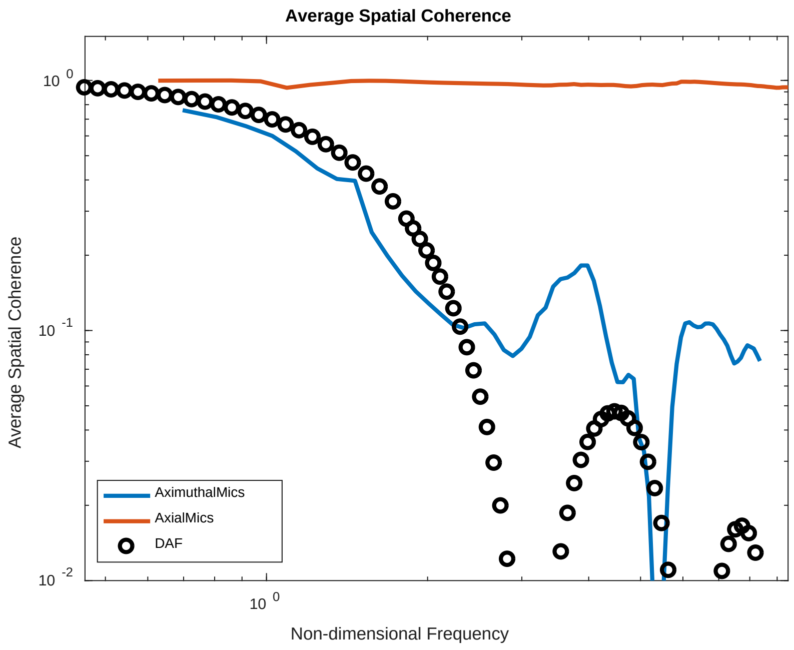
Split 2 Configuration

**All 15 stacks, split vertically into halves (Up to 30 independent inputs)*

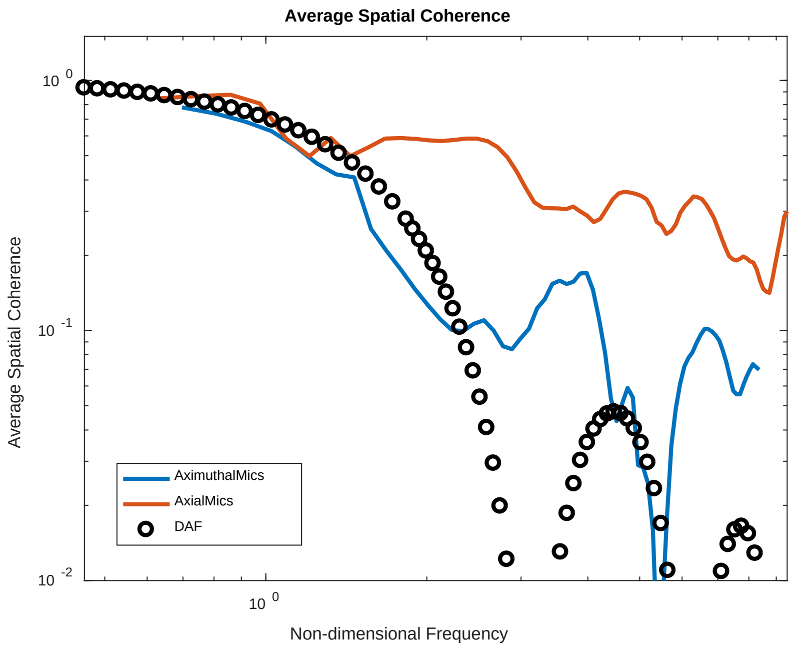


Random Uncontrolled Input– Spatial Coherence

No Split



Split 2



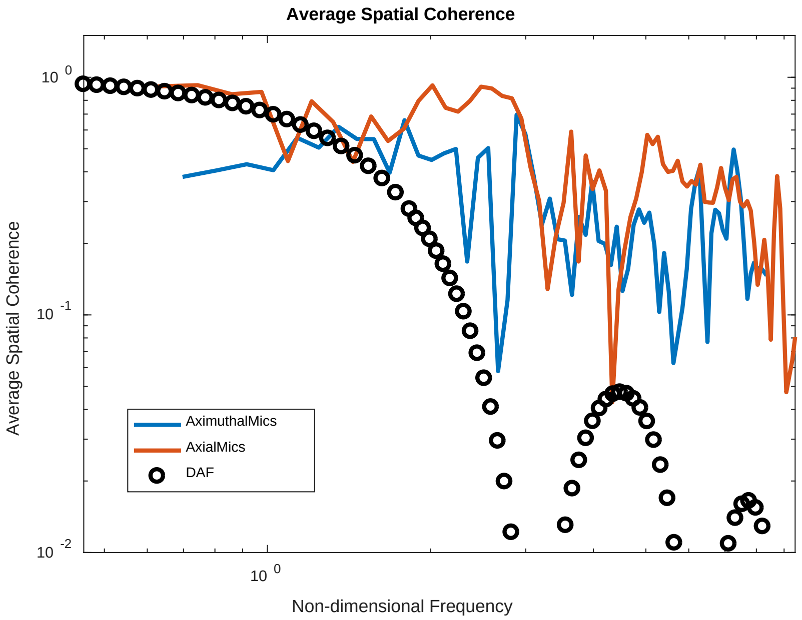
Direct Field

Split 2 configuration reduced axial coherence as predicted

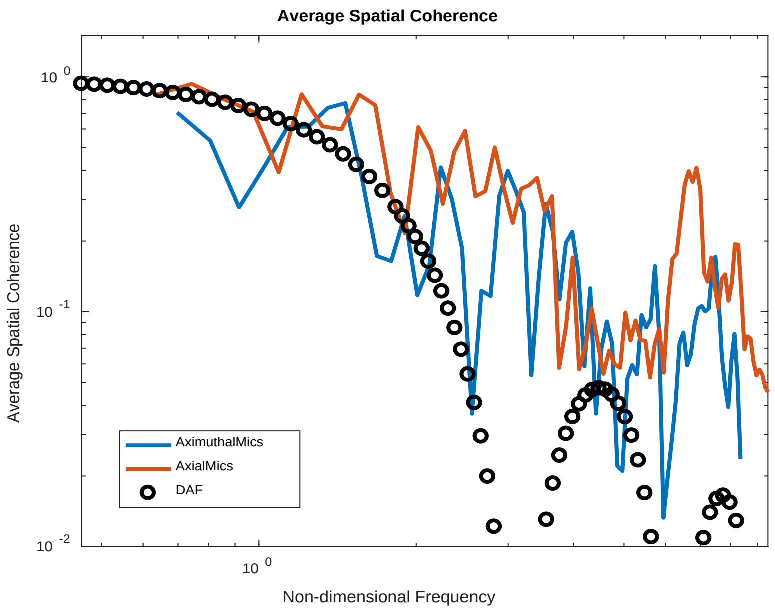
Random Uncontrolled Input– Spatial Coherence

Scatter

No Split



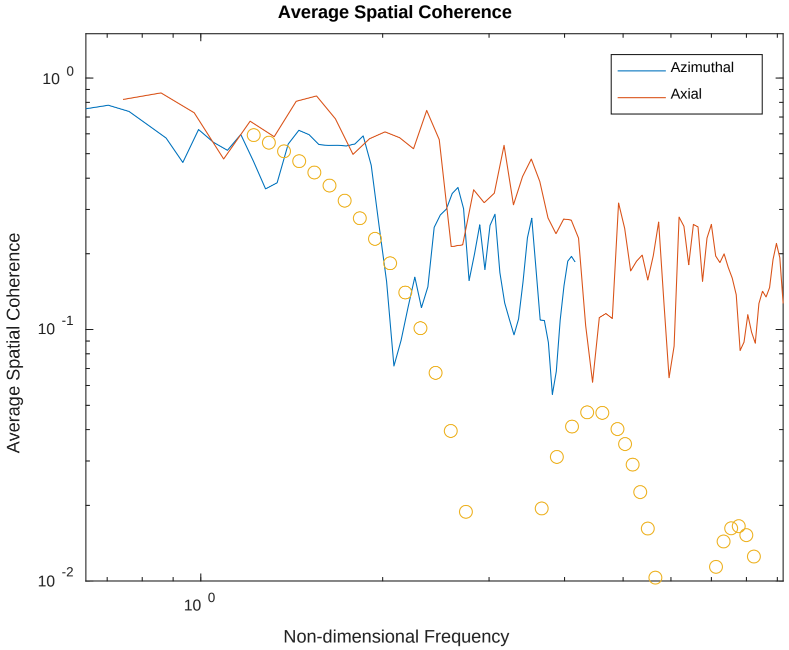
Split 2



Split 2 configuration reduced axial coherence as predicted

Split 2 – Alternate Input Configuration Random Uncontrolled Input

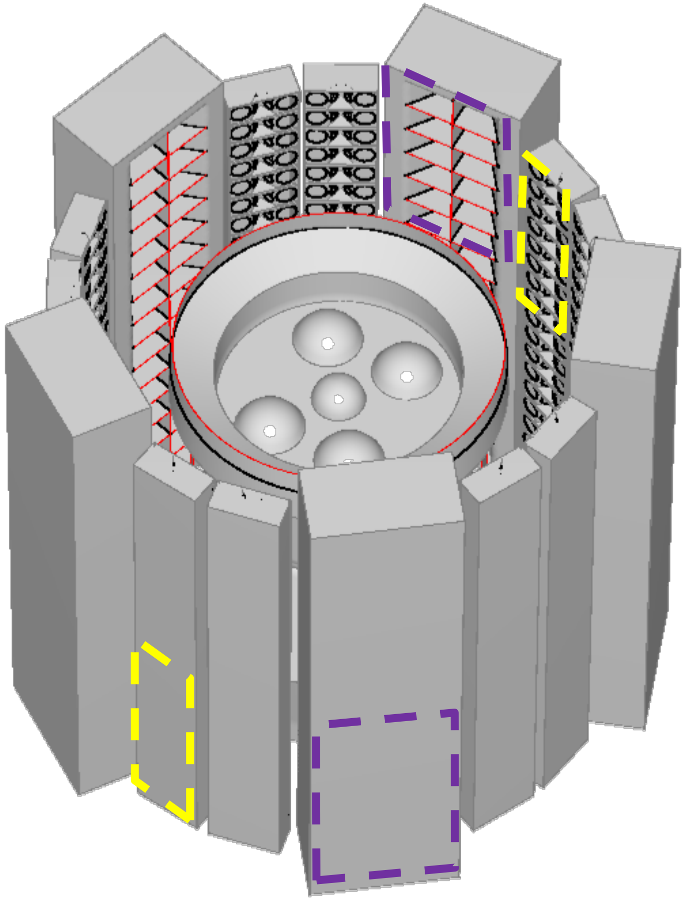
- Reduce independent inputs from 30 to 15:
  - 15 independent inputs
  - Independent inputs are not vertically adjacent



**Reducing the number of independent inputs does not significantly affect the cross spectrum results*

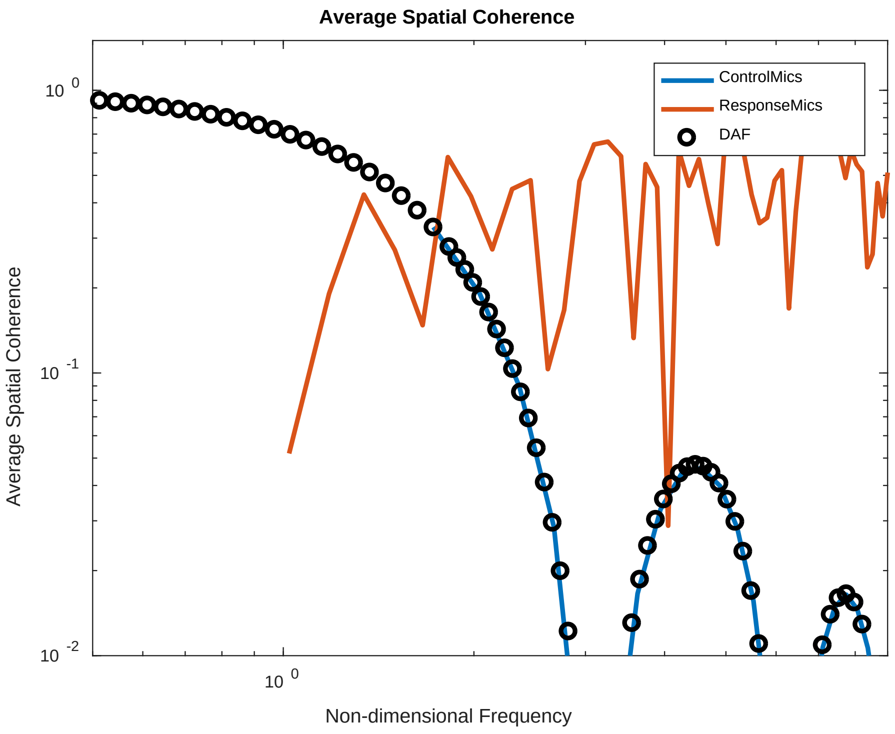
100% Correlated Woofer / Mid Stacks

100% Correlated Sub Stacks

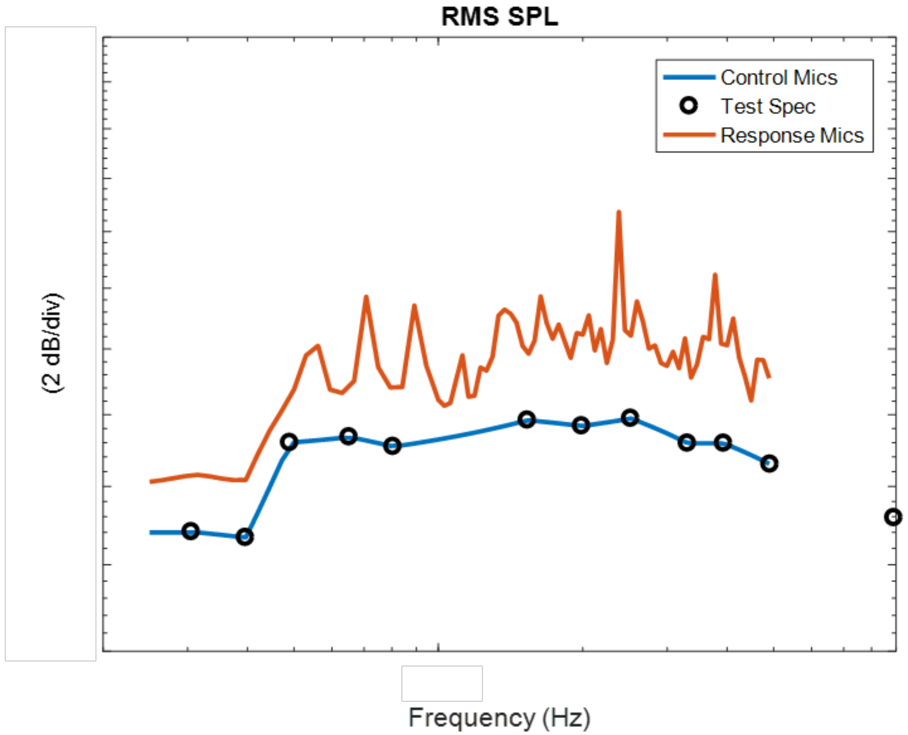


15 X 15 Control Simulation

$$\mathbf{G}_{vv} = \mathbf{H}^{-1}\,\mathbf{G}_{pp}\left(\mathbf{H}^{*T}\right)^{-1}$$



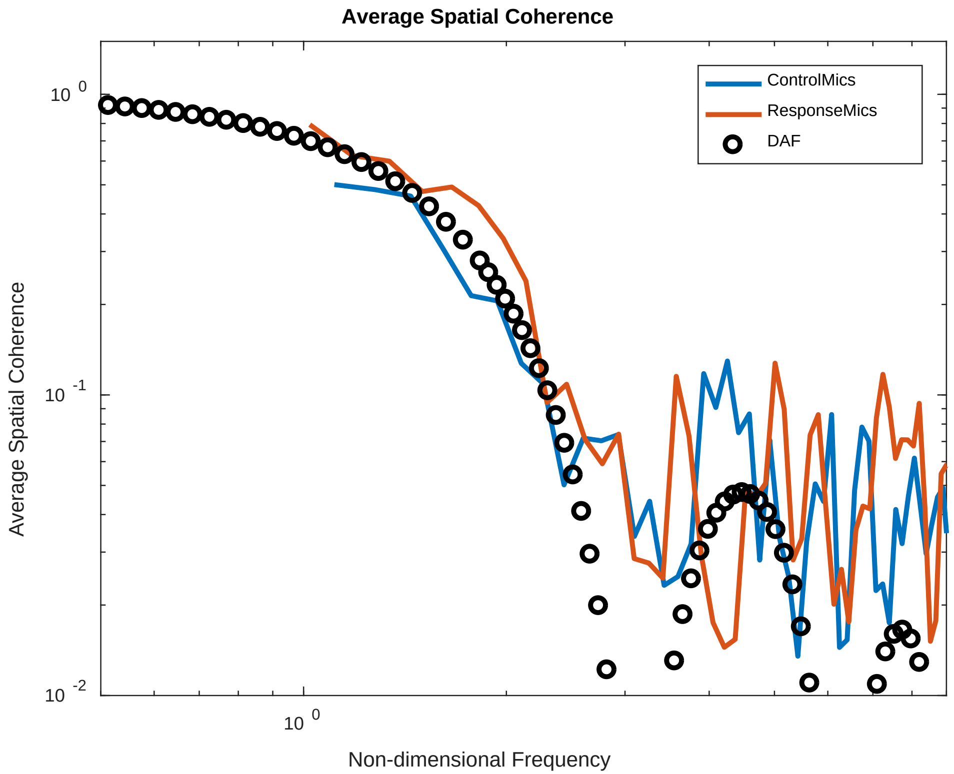
Control mics are diffuse, but response mics are not



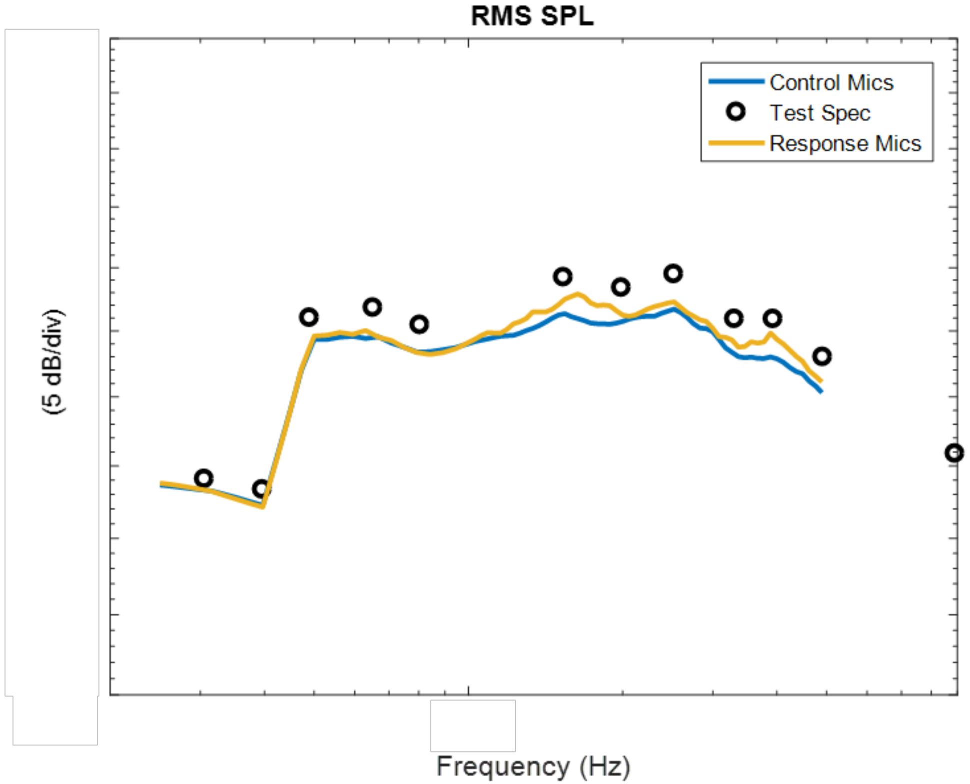
Control mics meet SPL requirement, but response mics are significantly louder

15 X 30 Control Simulation

$$G_{vv} = \left(H^T H\right)^{-1} H^T G_{pp} H^* \left(H^* H^{*T}\right)^{-1}$$



Control mics and response mics are an approximation of DAF



Control mics and response mics are within 3 dB of test spec SPL

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What we learned

1. MIMO Random Control can achieve "Ideal DAF" <u>but only at control mics</u>
   - <u>NOT at other locations; leading to spatial non-uniformity (up to +10 dB over drive)</u>
   - Controller target $G_{pp}(j\omega)$ should be based on in-situ measured (scattered) cross spectrum with multiple statistically independent inputs

2. Numerical (BEM) simulation can predict non-DAF spatial correlation of complex, full scale test configurations

3. Simulations indicate DFAT vertical spatial correlation can be improved by:
   - Vertical split of loudspeaker banks

     AND / OR
   - Rectangular (vs square) MIMO random control

Questions ?